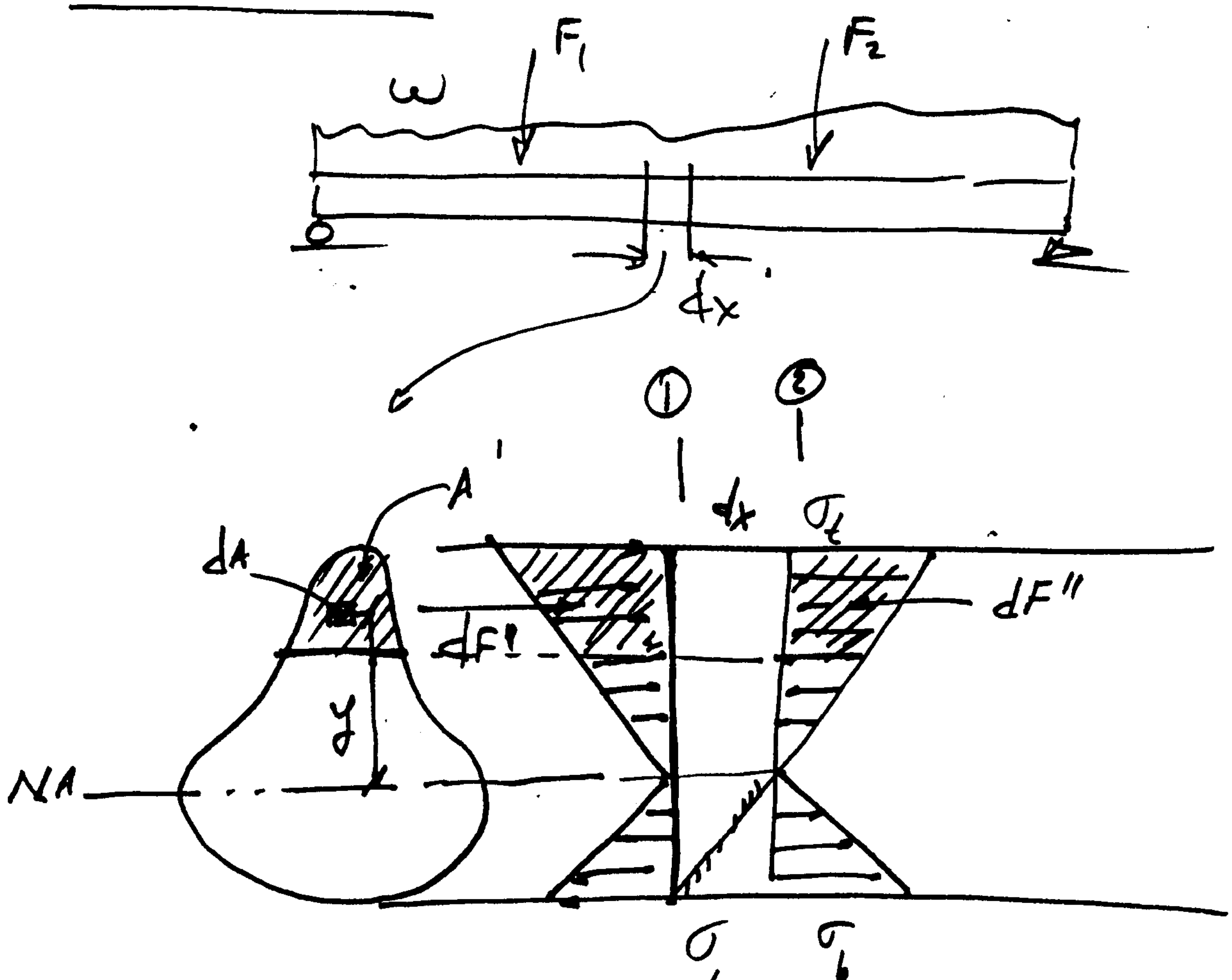
The shear formula

derivation:



section (1) the moment is M_1
section (2) The moment is M_2

$$\sigma_y = \frac{My}{I}$$

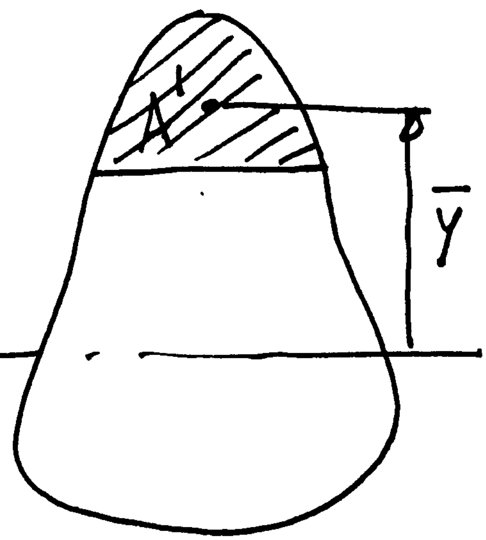
$$\therefore \quad dF' = \left(\frac{M_1 y}{I}\right) dA \qquad F' = \int_{A'} \frac{M_1 y}{I} dA$$

$$dF'' = \frac{M_2 y}{I} dA \qquad F'' = \int_{A'} \frac{M_2 y}{I} dA$$

OR V = shear force FROM V diagram

$$\frac{dM}{dx\,(I)(t)} \int_{A'} y\,dA = \tau$$

$$\tau = \frac{V}{It} \int_{A'} y\,dA$$



$$\int_{A'} y\,dA = \bar{Y}A' = Q$$

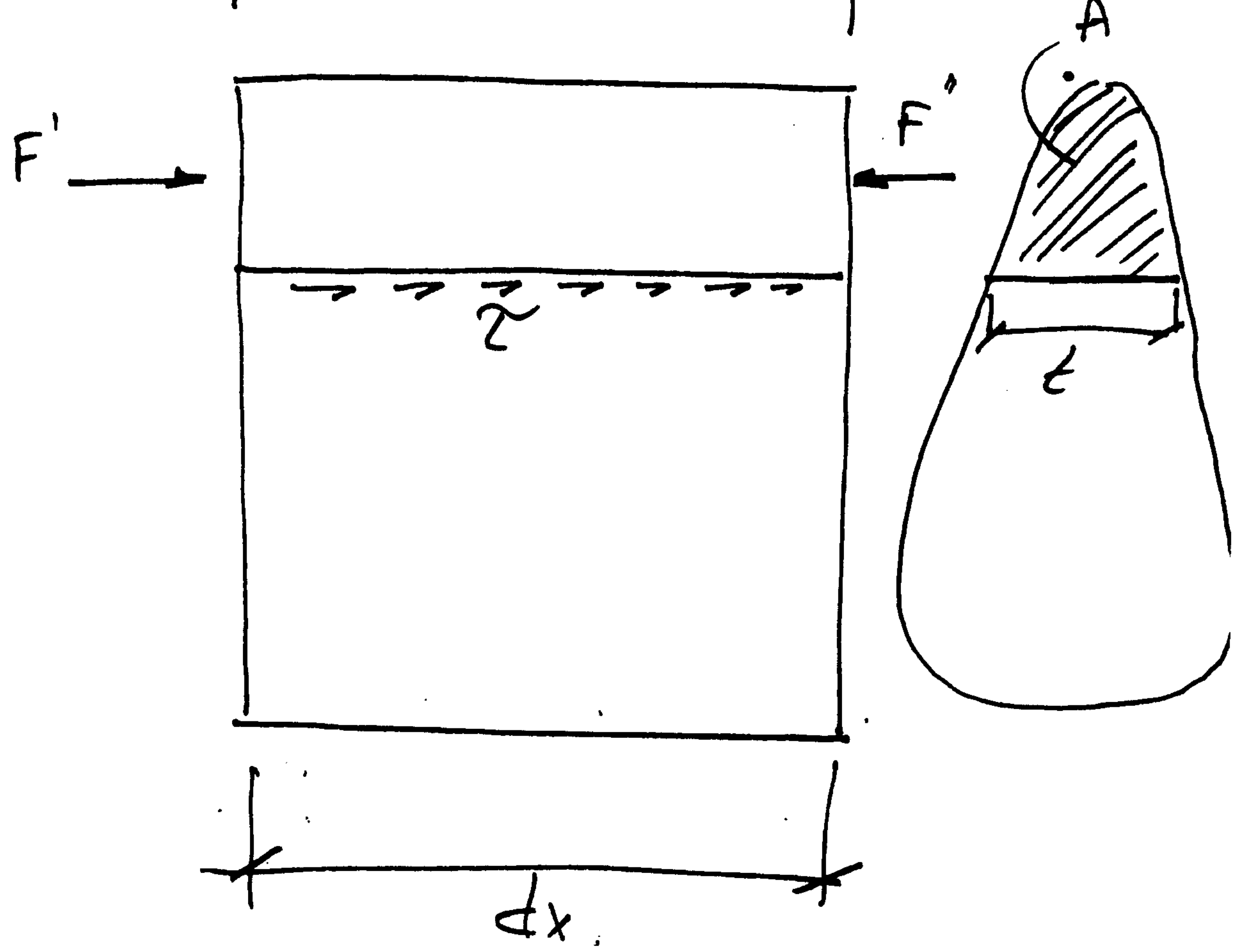
$$\tau = \frac{VQ}{It} \quad \text{where}$$

V = shear force at a section

Q = 1st Moment of A' about N.A.

I = Moment of inertia of the entire section

t = thickness of section where shear stress is computed.



We assume $M_2 > M_1 \therefore F_2'' > F_1'$

$\therefore$ STATICS of AREA A'

$$F' + \tau\, dx\, t - F_2'' = 0$$

$$\int_{A'} \frac{M_1 y}{I}\, dA - \int_{A'} \frac{M_2 y}{I}\, dA = -\tau\, dx\, t$$

$$\frac{M_2 - M_1}{I} \int_{A'} y'\, dA = \tau\, dx\, t$$

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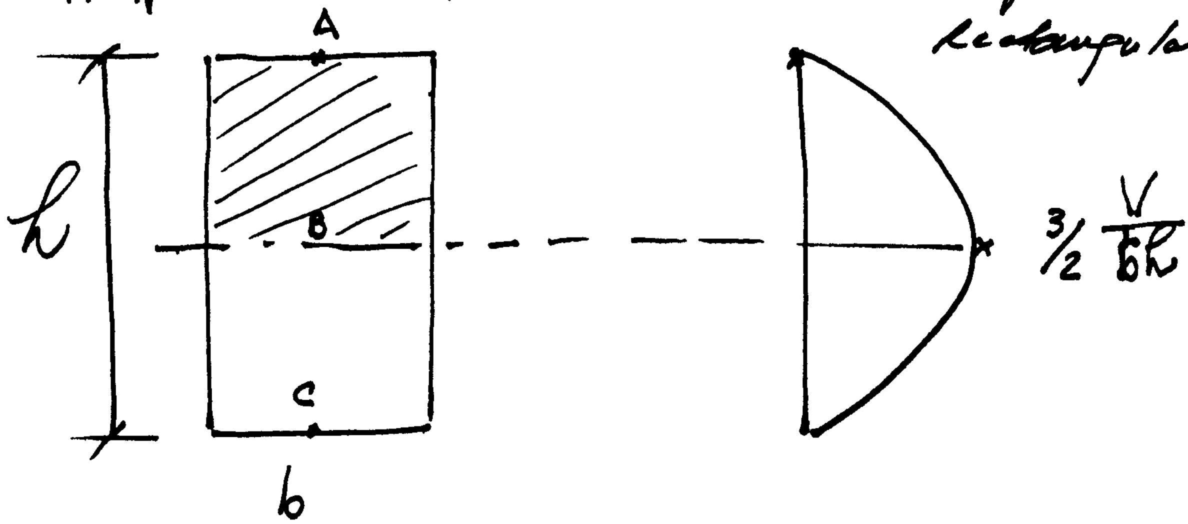
EXAMPLE: Plot τ stress distribution for a rectangular section.



at A $\quad Q = 0 \quad \therefore \tau = 0$

at B $\quad Q = b \cdot \frac{h}{2} \cdot \frac{h}{4} = \frac{bh^2}{8}$

$$I = \frac{1}{12} b h^3$$

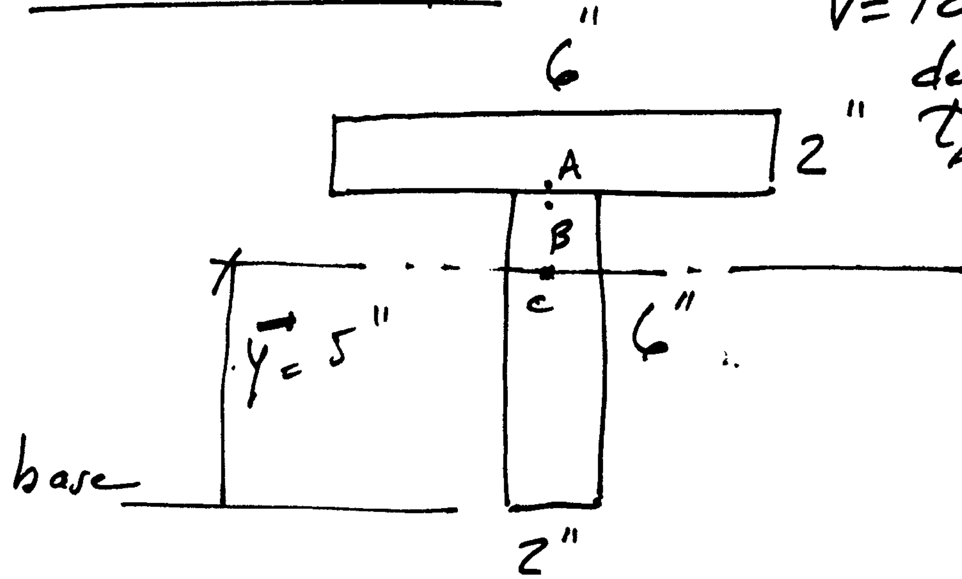
$$t = b$$

$$\therefore \tau = \frac{VQ}{It} = \frac{V \frac{bh^2}{8}}{\frac{bh^3}{12}(b)} = \frac{3}{2}\frac{V}{bh}$$

<u>another example</u> : $V = 10000$ lbs

determine τ_A, τ_B & τ_C

establish N.A.



$$(\bar{y})(24) = (12)(3) + (12)(7)$$

$$\bar{y} = 5''$$

$$I = \left(\frac{1}{12}\right)(2)(6)^3 + (12)(2)^2 + \frac{1}{12}\cdot 6 \times 2^3 + 12 \times 2^2$$

$$I = 136 \text{ in}^4$$

for A $\quad Q = (6 \times 2)(2) = 24 \text{ in}^3 \quad t = 6''$

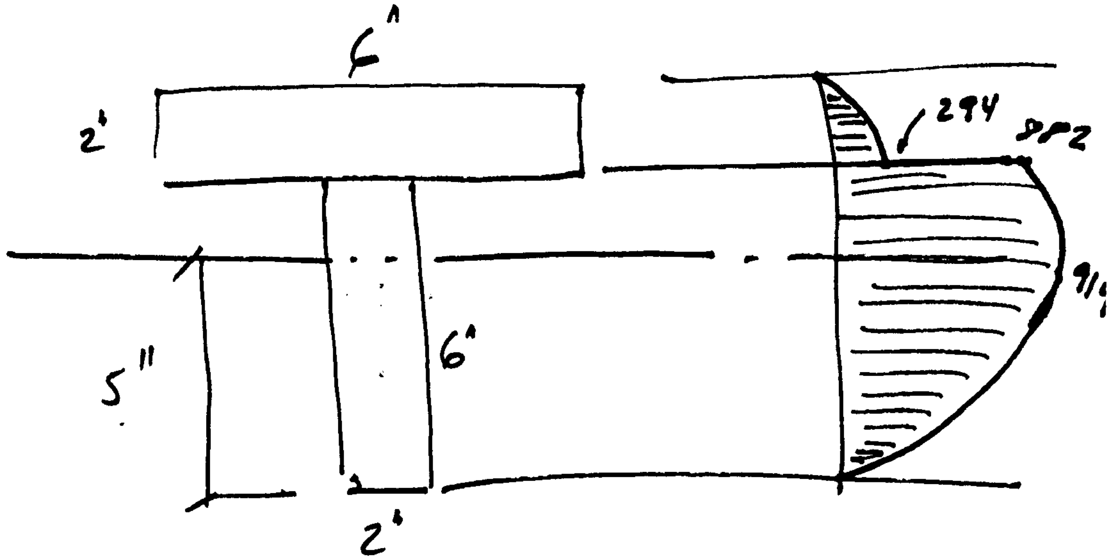
$$\therefore \tau_A = \frac{(10000)(24)}{(136)(6)} = 294 \text{ psi}$$

$$\tau_B = \frac{(10000)(24)}{(136)(2)} = 882 \text{ psi}$$

$\tau_C = ? \quad Q = (12)(2) + (1)(2)(0.5) = 25 \text{ in}^3$

$t = 2''$

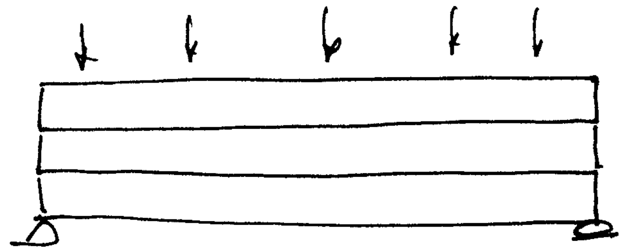
$$\tau_C = \frac{(10000)(25)}{(136 \times 2)} = 919 \text{ psi}$$

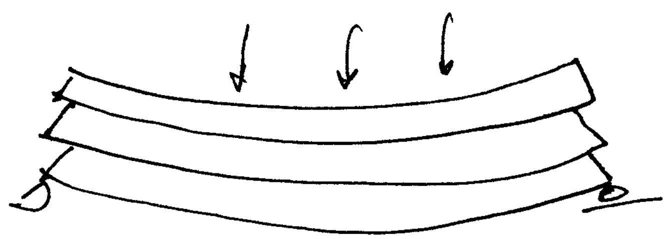


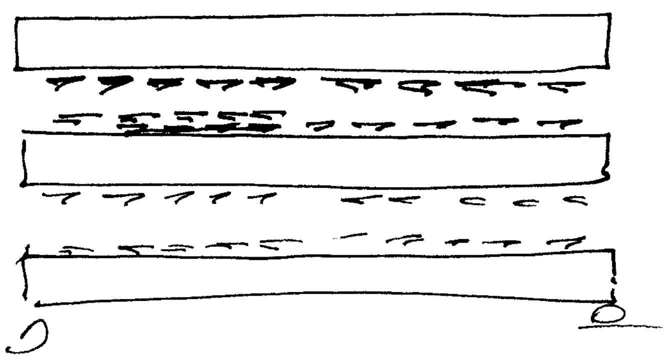
Results

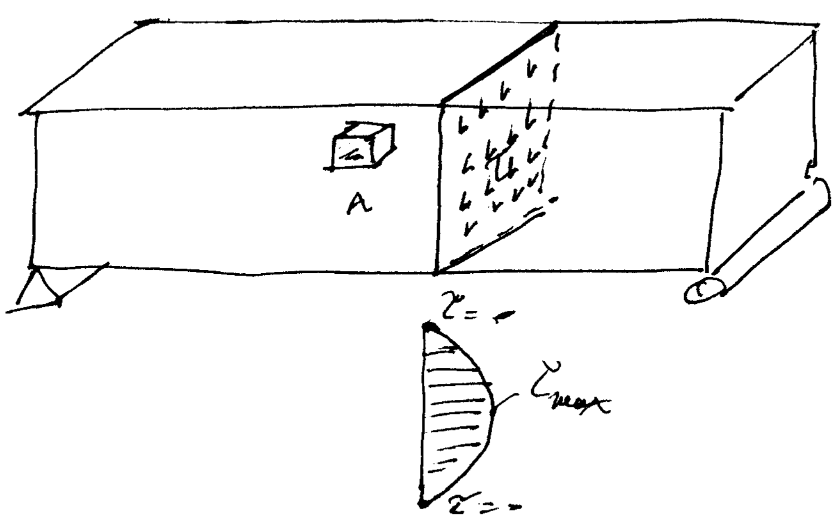
Lect 23

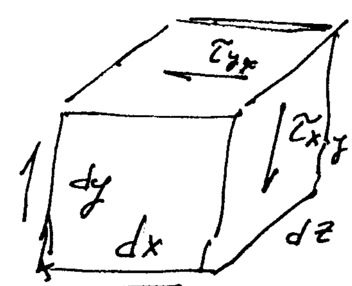
- Discuss Layered Beams





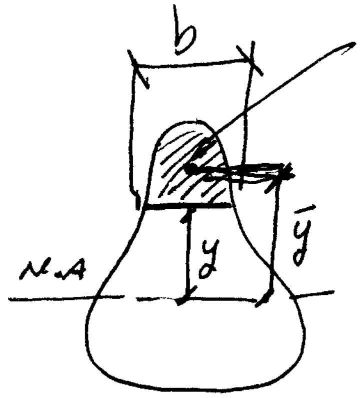




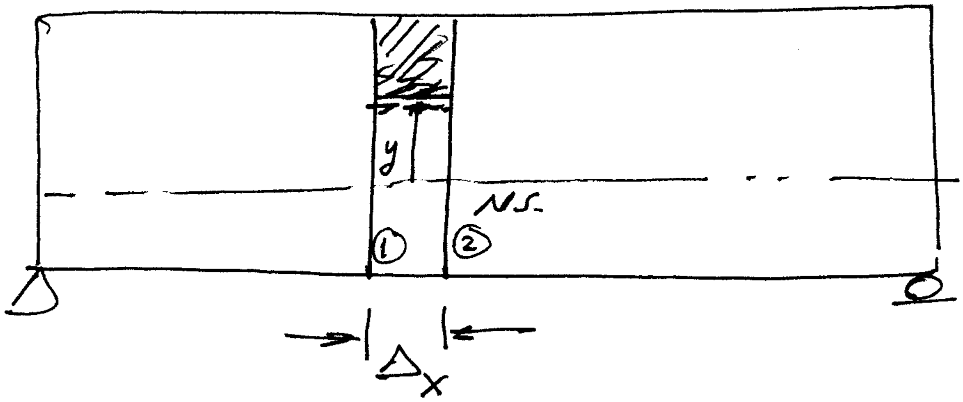


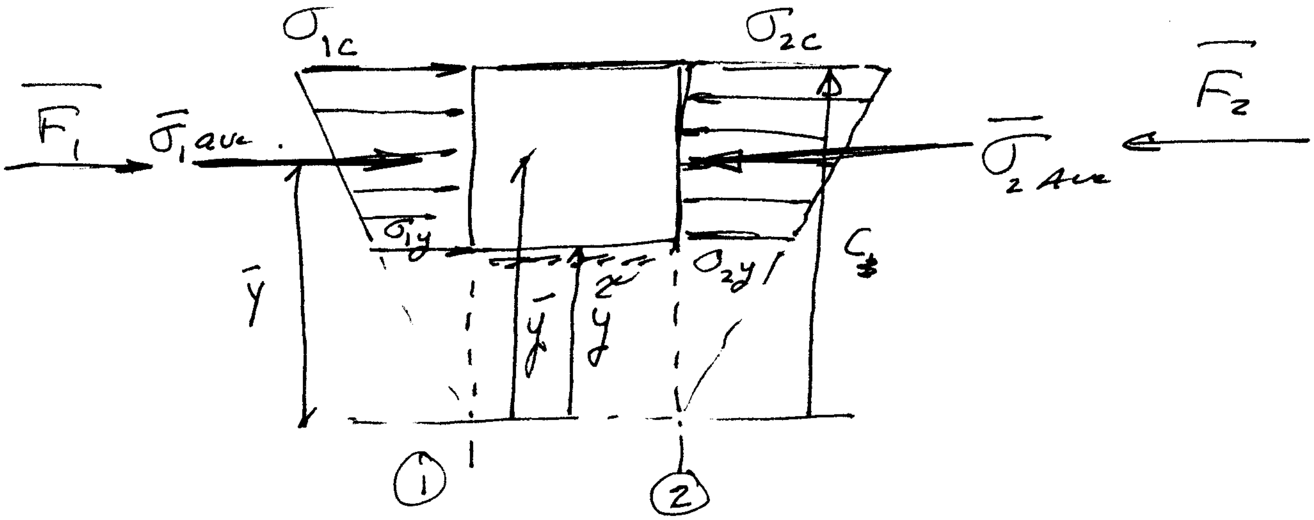
$$\tau_{xy}\, dy\, dz\, dx - \tau_{yx}\, dx\, dz\, dy = 0$$

$$\tau_{xy} - \tau_{yx} = 0$$



Cross section





$$\bar{\sigma}_{1ave} = \frac{M_1 \bar{y}}{I} \qquad \sigma_{2Ave} = \frac{M_2 \bar{y}}{I}$$

$$F_1 = \frac{M_1 \bar{y}}{I} dA \qquad F_2 = \frac{M_2 \bar{y}}{I} dA$$

dA = Area above the section where τ is to be computed.

Clearly $M_1 \neq M_2$ $\therefore$ $F_1 \neq F_2$

assume $M_2 > M_1$

$$F_1 + \tau \Delta x\, b - F_2 = 0$$

$$\frac{M_1 \bar{y}\, dA}{I} + \tau \Delta x\, b - \frac{M_2 \bar{y}\, dA}{I} = 0$$

$$\tau \, \Delta x \, b = \frac{M_2 \bar{y} \, dA}{I} - \frac{M_1 \bar{y} \, dA}{I}$$

$$\tau \Delta x \, b = \left(\frac{M_2 - M_1}{\cancel{I}}\right) \frac{\bar{y} \, dA}{I}$$

$$\tau = \frac{\Delta M}{\Delta x} \frac{\bar{y} \, dA}{I b}$$

$$\tau = \underbrace{\frac{dM}{dx}}_{V} \frac{\bar{y} \, dA}{I b} = \frac{VQ}{I b}$$

$$\boxed{\tau = \frac{VQ}{I b}}$$

V = Vertical shear force

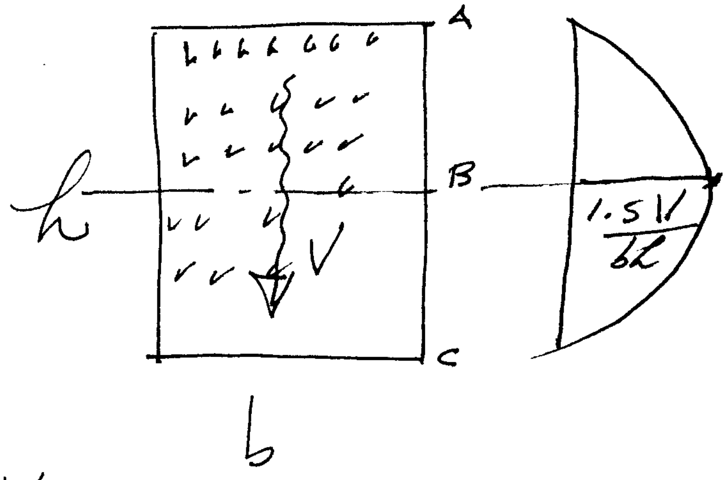
Q = First moment of the area above the location of desired τ about the N.A.

I = Moment of Inertia of the entire section

b = width of the cross section at the location of desired τ.

example: Given

V determine the shear stress distribution across the depth of the cross section



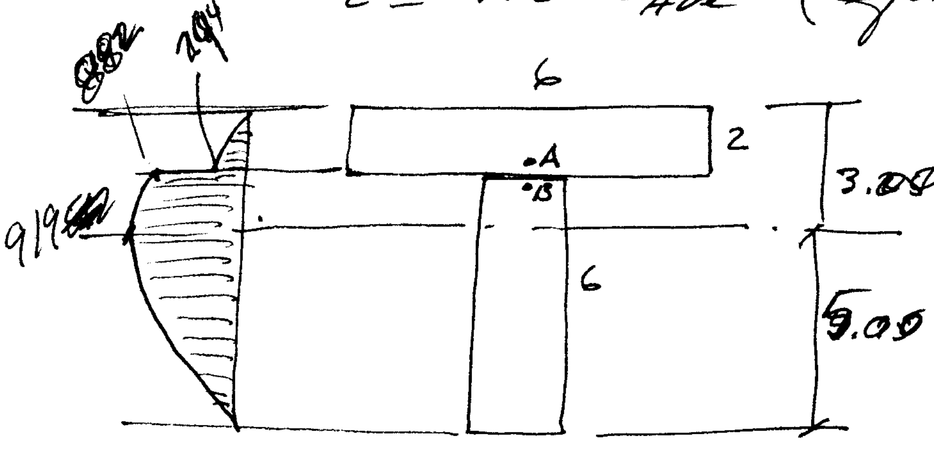
$$\tau_A = \frac{VQ}{Ib} = \frac{V(0)}{Ib} = 0$$

$$\tau_B = \frac{(V)(\frac{h}{2} \times b)(\frac{h}{4})}{\frac{bh^3}{12} \cdot b} = \frac{3V}{2bh} = 1.5\frac{V}{bh}$$

but $\frac{V}{\text{TOTAL Area}}$ = Average stress = τ_{ave}

∴ at the centroidal axis of the cross section

$\tau = 1.5\, \tau_{ave}$ (for a rectangular section)



V = 10 000 lbs.

τ @ A ?

τ = B ?

$I = \cancel{244}\ 136\ in^4$

b @ A = 6"

b @ B = 2"

$Q = (6 \times 2)(2) = 24\ in^3$

$$\tau_A = \frac{(10000)(24)}{(136)(6)} = 294\ psi$$

$$\tau_B = \frac{(10000)(24)}{136 \times 2} = 882\ psi$$

τ at centroidal axis $\quad Q = 10 \times 2.5$
$= 25$

$$\tau = \frac{(10000)(25)}{\cancel{288}\ 136 \times 2} = 919 \text{ psi}$$