SHOCK WAVES AND BOTTLENECKS

In the earlier discussion, vehicles were assumed to have roughly the same speed and their flow was treated in an aggregate manner. This section analyzes the effects of drastic changes in vehicle speed and describes how they progress throughout the traffic stream, hence the term "shock waves". The following discussion deals with only one lane and assumes that no passing is allowed.

Consider initially a traffic stream where speeds of individual vehicles vary slightly from the speed of the traffic stream, moving say at 40 mph. In this case, all vehicles in the traffic stream travel more-or-less at the same speed and hence nothing dramatic happens. Consider now that one vehicle slows down, (e.g., a truck on an up-grade or a passenger vehicle entering a speed zone) to say 10 mph. Since nobody can pass the slow vehicle, the entire traffic stream behind it must slow down to avoid a collision. This creates a shock-wave of slow-downs that, in this example, propagates upstream and results in the formation of a "platoon" or queue behind the slow-moving vehicle. Visualize the location of this shock wave as the last vehicle of the queue and note that this location will move with respect to the ground as more vehicles are added to the queue, (Figure 6.5).

![Figure 6.5: Shock Wave Propagation, (Papacostas, 1993).](image-url)
Its speed is defined as the derivative of flow rate \( q \) with respect to density \( k \), expressed as:

\[
U_w = \frac{dq}{dk} = \frac{q_B - q_A}{k_B - k_A}
\]  

(6.29)

where, \( A \) represents the conditions in the traffic stream before the slowdown, (i.e., flow \( q_A \) and density \( k_A \)), while \( B \) represents the conditions in the traffic stream in the queue, (i.e., flow \( q_B \) and density \( k_B \)). \( U_w \) is not to be confused with the speed of traffic either in the platoon or outside it is rather the speed of the last vehicle in the platoon with respect to the ground as shown in Fig. 6.6. Note that the speed of the shock wave can be either positive or negative depending on the relative flows and densities of the two traffic streams. If the speed is positive, the shock wave travels in the direction of traffic, (i.e., downstream) and vice-versa.

Figure 6.6: Definition of Shock Wave Speed \( U_w \)

Let us now expand on the earlier example: Say, that the slow moving vehicle at the front of the queue sped up again from 10 mph to 40 mph. Then, the platoon that was formed would be "released" and another shock wave will be created in the front of the queue. Say the traffic conditions in the queue are represented by \( B \), (as earlier), and the traffic conditions of the released vehicles are represented by \( C \). Then the speed of the shock wave in the front of the queue will be given by:

\[
U_w = \frac{q_C - q_B}{k_C - k_B}
\]  

(6.30)

Let us symbolize the speeds of the shock waves at the rear of the queue and the front of the queue as \( U_{w,A-B} \) and \( U_{w,B-C} \), respectively. Then the net speed of dissipation or build-up
of queue, $U_{NET}$ will be the **algebraic** difference between the speeds in the front and rear shock waves, given by:

$$U_{NET} = U_{W_{A-C}} - U_{W_{A-B}}$$  \hspace{1cm} (6.31)

Needles to say:
- if $U_{NET} < 0$ the queue will dissipate
- if $U_{NET} \geq 0$ the queue will not dissipate

This is illustrated in a q-k diagram as:

![Shock Wave Dissipation/Build-up](image)

**Figure 6.7: Shock Wave Dissipation/Build-up.**

**Example:**

A 20 mph school zone stays effective from 8-8:15 AM and causes a slow down of a traffic from conditions A to B. Subsequently, the traffic is released to traffic flow conditions C as specified below:

<table>
<thead>
<tr>
<th>Location</th>
<th>$q_i$</th>
<th>$v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approaching</td>
<td>$q_A = 1000$ veh/hr</td>
<td>$v_A = 40$ mph</td>
</tr>
<tr>
<td>Middle</td>
<td>$q_B = 1100$ veh/hr</td>
<td>$v_B = 20$ mph</td>
</tr>
<tr>
<td>Downstream</td>
<td>$q_C = 1200$ veh/hr</td>
<td>$v_C = 30$ mph</td>
</tr>
</tbody>
</table>

Determine:
- Speed and direction of the two shock waves
- Length of platoon being created
- Time required to dispense it

Densities of the three traffic streams:
\[
\frac{q_A}{v_A} = k_A = 25 \text{ veh/mi}
\]
\[
\frac{q_B}{v_B} = k_B = 55 \text{ veh/mi}
\]
\[
\frac{q_C}{v_C} = k_C = 40 \text{ veh/mi}
\]

\[
U_{wA\rightarrow B} = \frac{1100 - 1000}{55 - 25} = +\frac{100}{30} = +3.33 \text{ mph (i.e., moves downstream)}
\]

\[
U_{wB\rightarrow C} = \frac{1200 - 1100}{40 - 55} = -6.67 \text{ mph (i.e., moves upstream)}
\]

Growth of platoon = \(20 - 3.33 = 16.67 \text{ mph}\)

Length of platoon at the end of the 15 min period = \(16.67 \times \frac{1}{4} \text{ hr} = 4.17 \text{ mi}\)

Involving \(4.17 \times 55 = 230\) vehicles

Relative speed of 2 shock waves = front - rear

\[
U_{NET} = -6.67 - (+3.33) = 10 \text{ mph}
\]

Hence, it will take \(\frac{4.17 \text{ mi}}{10 \text{ mph}} = 0.417 \text{ hrs} = 25 \text{ min}\) to disperse

Continue example by drawing trajectories (space-time diagrams) for the 2 shock waves of the last example from the start of the 15-minute long speed reduction enforcement period until the platoon formed has dissipated, (Khisty, 1991):
This section describes how the minimum spacing between vehicles can be calculated and how this can be used to calculate safe $k$ and $q$ at various speeds.

Consider two successive vehicles in the traffic stream, lead and following and let's calculate the minimum spacing between them, (Figure 6.8).