Problem 1 (20 points) – The W12x87 steel column shown below is 12 ft. long and is fixed at the base and free at the top (for buckling about any axis of the cross-section). Determine the load that will cause failure of the column. Properties of the W12x87 column include:

\[ E = 29,000 \text{ ksi} \quad F_y = 50 \text{ ksi} \quad A = 25.6 \text{ in}^2 \quad I_x = 740 \text{ in}^4 \quad I_y = 241 \text{ in}^4 \]

**SOLUTION**

Failure Load = \[ \frac{754}{k} \]

\[ K_x = K_y = 2.1 \]

\[ l_x = l_y = 12 \text{ ft} \]

\[ I_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{740}{25.6}} = 5.37 \text{ in} \]

\[ I_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{241}{25.6}} = 3.06 \text{ in} \]

\[ P_E = \frac{\pi^2 E}{(KL)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{\left(\frac{2.1(12 \text{ ft})(12 \text{ in/ft})^2}{3.06^2 \text{ in}}\right)^2} = 2947 \text{ ksi} \leq F_y \]

\[ P_E = F_E A = (29,471 \text{ ksi})(25.6 \text{ in}^2) = 754.3 \text{ k} \]

Alternatively,

\[ P_E = \frac{\pi^2 E}{(KL)^2} \left[\frac{I_y}{A}\right]_{\text{min}} = \frac{\pi^2 (29,000 \text{ ksi})(241 \text{ in}^4)}{(2.1)(12 \text{ ft})(12 \text{ in/ft})^2} = 754.3 \text{ k} \]
Problem 2 (30 points) – The compound beam shown below has a roller support at Point A, a fixed support at Point C, and an internal hinge at Point B. Below the FBD of the compound beam, provide a complete shear diagram and a complete moment diagram. You may ignore the self-weights of the members. On the shear and moment diagrams you must indicate the magnitudes of shear (or moment) at each location where there is an abrupt change in slope or magnitude on the diagram.

[NOTE that the external reactions have already been determined at the fixed support and roller.]
Problem 3 (20 points) – The beam shown below is supported by a pin at Point B and a roller at Point C. External reactions at Points B and C have been determined, and are shown on the FBD. The shear and moment diagrams for the beam are also provided. The arbitrary distance $x$ is defined as positive to the right, with $x = 0$ at Point A. Determine:

A) **The equation for moment as a function of $x$ in Region BC.**  
   [i.e.- determine $M(x)$ when $3 \ m \leq x \leq 9 \ m$]

B) **The magnitude of maximum moment ($M_{\text{max}}$) in Region BC.**

**SOLUTIONS**

A) $M(x) = -\frac{x^3}{2} + 60.75x - 182.25$

$$w = \frac{27 \text{ KN/m}}{3 \text{ m}} = 9 \text{ KN/m}$$

B) $M_{\text{max}} = \frac{75.5 \text{ kN} \cdot \text{m}}{w = 9 \text{ KN/m}}$

$$+\sum M_{\text{about A}} = 0: M = 60.75(x-3) + \frac{1}{2}(3x)(x)(\frac{x}{3}) = 0$$

$$M = -\frac{x^3}{2} + 60.75x - 182.25$$

**in Region BC**

$$3 \ m \leq x \leq 9 \ m$$

$$V(x) \text{ (kN)}$$

$$M_{\text{max}} = -\frac{(6.36)^3}{2} + 60.75(6.36) - 182.25$$

$$= 75.5 \text{ kN} \cdot \text{m}$$
\[ \sum M_{at} = 0: (60.75)(9-x) - M - (3x)(9-x)\left( \frac{9-x}{2} \right) - \frac{1}{2}(9-x)(27-3x)(\frac{3}{2})(9-x) = 0 \]

\[ M = 546.75 - 60.75x - \frac{3}{2}(9-x)^2 - (9-x)^3 \]

\[ M = 546.75 - 60.75x - 121.5x + 27x^2 - \frac{3}{2}x^3 - 729 + 162x \]

\[ -9x^2 + 51x - 18x^2 + x^3 \]

\[ M = -\frac{x^3}{2} + 60.75x - 182.25 \quad \text{in region BC} \]
Problem 4 (20 points) – The beam shown below is supported by a pin at Point A and a roller at Point D. External reactions at Points A and D have been determined as: $A_y = 46\ k$ (upward) and $D_y = 54\ k$ (upward). The shear and moment diagrams for the beam are also provided.

**Determine the location of the inflection point in Region CD** (i.e.- determine the location where $M = 0$).

**SOLUTION**

The inflection point is located $6.75\ ft$ to the right of Point C.

\[ M = -x^2 - 14x + 140\ ]

in Region CD ($0 \leq x \leq 10\ ft$)

\[ \sum M_{eq} = 0: M + (2k)(x)(\frac{x}{2}) + 180 + 60(x+10) - 46(x+20) = 0 \]

\[ M = -x^2 - 14x + 140\ ]

\[ \phi = (x^2 + 14x + 49) - 189 \]

\[ (x+7)^2 = 189 \]

\[ x = -7 \pm \sqrt{189} \]

A negative distance would not be in the region $0 \leq x \leq 10\ ft$

\[ \therefore x = -7 + \sqrt{189} = 6.75\ ft \]
**Problem 4** - Alternate method using areas from shear diagram

\[ V(x) \]

\[ \frac{1}{2} \cdot 20 \cdot x = -2x \]

\[ -10 \text{ ft} \rightarrow \rightarrow \text{10 ft} \rightarrow \rightarrow \rightarrow \rightarrow x \rightarrow \]

Set \( S_{(\text{areas})} = 0 \):

\[ (46)(10) + (-14)(10) + (-14)(x) + \frac{1}{2}(-2x)(x) - 180 = 0 \]

Remember to consider the couple at C

\[ 140 - 14x - x^2 = 0 \]

\[ x = -7 \pm \sqrt{189} \]

Same as previous solution

\[ x = -7.75 \text{ ft to the right of Point C} \]
Problem 5 (10 points) – The frame shown below supports a horizontal point load of 20 kN at rigid connection B. The reactions at Point A and Point D are both assumed to be pin supports, and the connection at Point C is assumed to be an internal hinge. Provide a sketch of the deflected shape of Frame ABCD.

[NOTE that FBDs have been provided for all members, and shear and moment diagrams have been provided for Member AB and Member BC. See next page.]

[Diagram of frame with labels: B, C, A, D, 20 kN load, 8 m, 10 m, internal hinge, pin support, entire frame sways to the right, AB is concave to the left (due to positive moment), BC is concave upward (due to positive moment), CD remains perfectly straight (no concavity), since no internal moment, connection at B maintains 90° angle, frame stays attached to pin supports at A and D, angle between BC and CD at internal hinge is < 90°, Point B and Point C must each displace laterally the same amount]
Problem 5 (continued)

\[ M_{bc} = 160 \text{ kN}\cdot\text{m} \]

\[ V_{bc} = 16 \text{ kN} \]

\[ V_{cb} = 16 \text{ kN} \]

\[ M_{ba} = 160 \text{ kN}\cdot\text{m} \]

\[ N_{ba} = 16 \text{ kN} \]

\[ V_{ba} = 20 \text{ kN} \]

\[ A_x = 20 \text{ kN} \]

\[ A_y = 16 \text{ kN} \]

\[ D_y = 16 \text{ kN} \]

\[ N_{c3} = 16 \text{ kN} \]