OPEN SECTIONS - THIN WALL THEORY:

\[ \phi = \frac{\theta}{L} \]
\[ \phi = \frac{M}{JG} \]
\[ J = \frac{6t^3}{3} \]
\[ \varepsilon = \frac{M}{6t^3} \]

APPROACH FROM PLATE THEORY:

\[ M_{xy} = D(1-\nu)\frac{w_{xy}}{t^2} \]
\[ t = \frac{C M_{xy}}{t^2} \]
\[ w = C \cdot xy \]
\[ w_y = Cx \]
\[ @ x = 0 \quad w_y = 0 \]
\[ C_L = \theta \]
\[ \phi = \frac{\theta}{L} = \theta \]
\[ M_{xy} = \frac{E h (1 - \nu)}{12(1 - \nu^2)} \phi = \frac{E h^3}{12(1 - \nu^2)} \phi \]

For isotropic materials, \( G = \frac{E}{2(1 + \nu)} \)

\[ M_{xy} = \frac{G h}{6} \phi \]

If we replace \( M_{xy} \) with a couple force \( 2M_{xy} \theta_b \)

\[ M = 2M_{xy} \theta_b = \frac{G h^3}{12} \phi \]

\[ J = \frac{3}{h} \]

\[ M = GJ \phi \]

\[ \phi = \frac{M}{GJ} = \text{same twist as determined from elasticity} \]

So plate theory indicates twist is resisted by small couples.

\[ \varepsilon_{\max} = \frac{6 M_{xy}}{h^2} \]

\[ \varepsilon_{\max} = \frac{6}{h^2} \frac{M}{26} = \frac{3M}{6h^2} \] (Same result as found from elasticity)

Note: \( \varepsilon \neq \phi \) are from plate bending now, not from in-plane direct stress (membrane)
In closed sections, \( I \) & \( \phi \) are from in-plane (direct) stresses.

Closed section stress distribution:

[Image of closed section with labeled arm = size of box]

In open sections, \( I \) & \( \phi \) are from plate bending.

Open section stress distribution:

[Image of open section with labeled formulas]

Now:

\[
M = M_1 + M_2 = (J_1 G_1 + J_2 G_2) \phi
\]

If \( G_1 = G_2 \),

\[
M = (J_1 + J_2) G \phi = \left( \frac{b_1 t_1^3}{3} + \frac{b_2 t_2^3}{3} \right) G \phi
\]

\[
J = \sum_{i=1}^{n} \frac{b_i t_i^3}{3}
\]
NOTE: JOINTS DO NOT AFFECT THE RESULTS, THEREFORE DO NOT WORRY ABOUT THEM. CURVES CAN BE CUT INTO MULTIPLE STRAIGHT SECTIONS

\[ M = J G \phi \]

\[ n = 7 \]

\[ J = \frac{\pi}{2} \frac{b_i t_i^3}{3} \]

\[ M_5 = \frac{J_5}{J} M \]

\[ \varepsilon_5 = \frac{M_5}{b_5 \frac{t_i^3}{3}} \]
End Diaphragms are required to transfer torsion.

Two options to attach:

Weld \( M_x \) to ARM.

If both sides are welded, the transfer arm is larger thereby reducing the transfer stress.

For thicker walled sections,

\[
\frac{b t^3}{3} = \frac{b t^3}{3} \left(1 - 0.083 \left(\frac{t}{b}\right)^2\right)
\]

However for \( \frac{t}{b} \leq \frac{1}{5} \), use \( \frac{b t^3}{3} \).

\( G \neq \frac{E}{2(1+\nu)} \) is the reason.

Let's re-investigate a closed box.

\[
M = J_{\text{closed}} G \Phi + J_{\text{open}} G \Phi
\]

\[
J_c = \frac{4A^2}{3} = \frac{4b^4}{4b^4} = \frac{t b^3}{3}
\]

\[
J_o = \varepsilon \frac{b t^3}{3} = \frac{4 b t^3}{3}
\]

\[
J = \frac{2 b^3}{3} + \frac{4 b t^3}{3}
\]
\[ J = \frac{\sqrt{6}}{6} \left[ 1 + \frac{4}{3} \left( \frac{t}{6} \right)^3 \right] \]

This overestimates by \( \frac{4}{3} \left( \frac{t}{6} \right)^2 \)

For \( t = \frac{1}{2}'' \), \( b = 5'' \)

\[ J = \frac{\sqrt{6}}{6} \left[ 1 + \frac{4}{3} \left( \frac{1}{100} \right) \right] = \frac{\sqrt{6}}{6} \left[ 1 + 0.01333 \right] \]

Use this if the open section is large relative to the closed section.

Suppose the box is the core of a high rise.

[Diagram of a box with dimensions and annotations]
ANALYZE AS A CANTILEVER.

\[ \Delta = \Delta_b + \Delta_v \]

\[ \Delta = \frac{VL^3}{3EI} + \frac{VL}{AG} \]

\[ \Delta = \frac{120q(50)^3}{3E(8)(30)^3} + \frac{120q(50)}{(8)(30) G} \]

IF WE WERE TO SET THIS EQUAL TO THE DEFLECTION OF A CLOSED SECTION OF THICKNESS \( \varepsilon \)

\[ \Delta_{sh} = \frac{qL^3}{6E} (50) \]

\[ \Delta = \Delta_{sh} \Rightarrow \varepsilon = 0.3'' \]

OPEN THEORY IS CALLED ST. VENMANT TORSION